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December 22, 2010

Diminished Triads and Scale Networks in the Hexagonal Virtual Room

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INTRODUCTION

In my previous paper entitled "Using Geometric Models to Compose in Virtual Realms," I imagined a circular room in virtual space, divided into six wedges and surrounded by six voices, with each individual wedge hearing only three of these voices: the one at its peripheral edge and the two to its immediate left and right. I showed how Jack Douthett's Cube Dance[1] may be used as a compositional space for this "hexagonal" virtual room,[2] creating ten different permutations for the arrangement of pitch-class intervals between the six voices such that the three voices heard within each wedge constitute a consonant or augmented triad. Additionally, the triads heard by adjacent wedges are always related by parsimonious voice-leading, thus ensuring a smooth musical transition as the listener wanders around the room.

The present paper continues to build upon this idea in four main sections. First, I now allow for the possibility of diminished triads, increasing to eighteen the total number of triadic permutations possible for the hexagonal room. I show that these may be visualized by extending the voice-leading already present within the Cube Dance to include diminished triads. Second, I examine how Dmitri Tymoczko's scale lattice for seven-note Pressing scales[3] and his work on the interrelation between voice-leading and key signatures[4] may be combined into scale and key signature charts. These charts may then be used by a composer to ensure that the three voices heard within each wedge in the virtual room are always confined to a single scale at any given time. Third, I show how a coherent compositional design may be constructed using the eighteen triadic permutations and the scale and key signature charts. I then demonstrate some ways in which a compositional design might be realized into a musical piece, using a brief example of my own. Fourth, I suggest some areas for future research, including the possibility of a virtual room in which triads, sevenths, and chords of greater cardinality might be connected by way of Clifton Callender's split and fuse transformations.[5]

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[1] Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *Journal of Music Theory* 42, no. 2 (Autumn 1998): 253-54.
[2] Although I envision the virtual room as a circle in its physical layout and have drawn it as such, I will refer to its six-wedge formation as a "hexagon." This designation will prove judicious in the fourth section of this paper, when I explore the possibility of rooms divided into eight or more wedges.
[3] Dmitri Tymoczko, "Scale Networks and Debussy," *Journal of Music Theory* 48, no. 2 (Fall 2004): 239-240.
[4] Dmitri Tymoczko, "Voice Leadings as Generalized Key Signatures," *Music Theory Online* 11, no. 4 (October 2005): 1-44.
[5] Clifton Callender, "Voice-Leading Parsimony in the Music of Alexander Scriabin," *Journal of Music Theory* 42, no. 2 (Autumn 1998): 224-32.

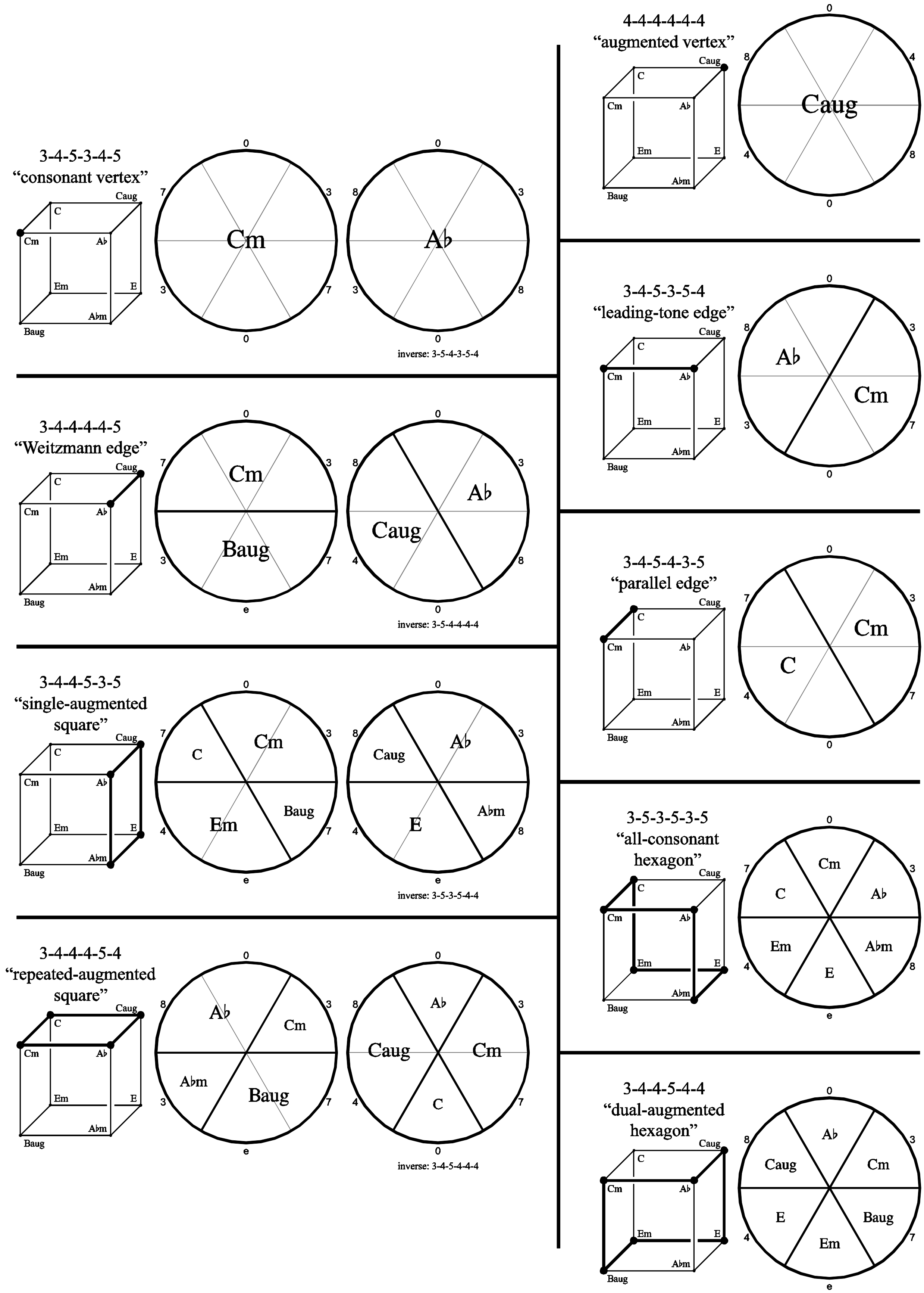

Figure 1: Nine permutations within the hexatonic cube

I. Diminished Triads in the Hexagonal Room

For any given "snapshot in time," each of the six voices surrounding the hexagonal room raises the pitch-class of the preceding voice by an interval-class of 3, 4, or 5, moving either in a clockwise or counter-clockwise direction. Naturally, these six interval classes must always span a double octave. In "Using Geometric Models," I excluded the placement of consecutive ic3s and ic5s to ensure that every possible distribution of interval classes for the hexagonal room under these guidelines, with one exception, can be isomorphic to a closed path within the Cube Dance. There are ten such permutations, nine of which are shown in Figure 1.

In Figure 1, each permutation is listed with a descriptive label, and the clockwise order of interval-classes between each voice is arranged in prime form. The pitch classes heard by each voice are written as mod 12 integers, beginning with pc0 in the northern voice. The top four permutations are asymmetrical and are shown with their inverse counterparts; in each case, only the prime form arrangement, and not its inverse, is shown placed within the hexatonic cube. A notable characteristic of these nine permutations is that opposite voices always differ at most by a semitone. The lone exception, which is not included in Figure 1 and which does not fit into the Cube Dance, is 3-4-3-5-4-5, which I call the "relative edge" as it divides the room between relative triads. Intuitively, it contains opposite voices which differ by a whole tone, reflecting the neo-Riemannian R operation.

From a musical perspective, of course, a system which contains only consonant and augmented triads is needlessly restrictive. By including ic6 and allowing for consecutive ic3s, we greatly increase our options with diminished triads. This results in eight more room permutations in which every set of three consecutive voices must form either a consonant, augmented, or diminished triad, bringing the total number to eighteen. These eight new permutations, along with the "relative edge" permutation, are shown in Figure 2, which is similar to Figure 1 in its underlying features. Each of the permutations in Figure 2 may be visualized within a simplified extension of the Cube Dance resembling a "billfold," which I shall now explain.

Figure 3 shows how two adjacent hexatonic cubes of the Cube Dance can be seen as part of a 2x2x2 cube, one in which parallel segments directly across from each other represent the same voice-leading paths.[6] Diminished triads thus come into being by simply continuing along these paths to create new vertices, as shown in Figure 4. For example, moving from B to C and from F♯ to G takes B-aug to C-dim. Within each of these hexatonic 2x2x2 cubes, there are three diminished triads that may be formed in this manner. Figure 4 then proceeds to show that all the possible voice-leading paths connecting every adjacent triad found in the eight new permutations, including whole tones and a minor third separating two diminished triads, combine to form a single "billfold" graph centered around just one of these diminished triads. Each of the eight new

[6] Dmitri Tymoczko, "Which Graphs Can We Trust?", (paper presented at the annual meeting of the Society for Music Theory, Indianapolis, IN, November 7, 2010). Tymoczko points out that voice-leading relationships can always be represented as cubes.

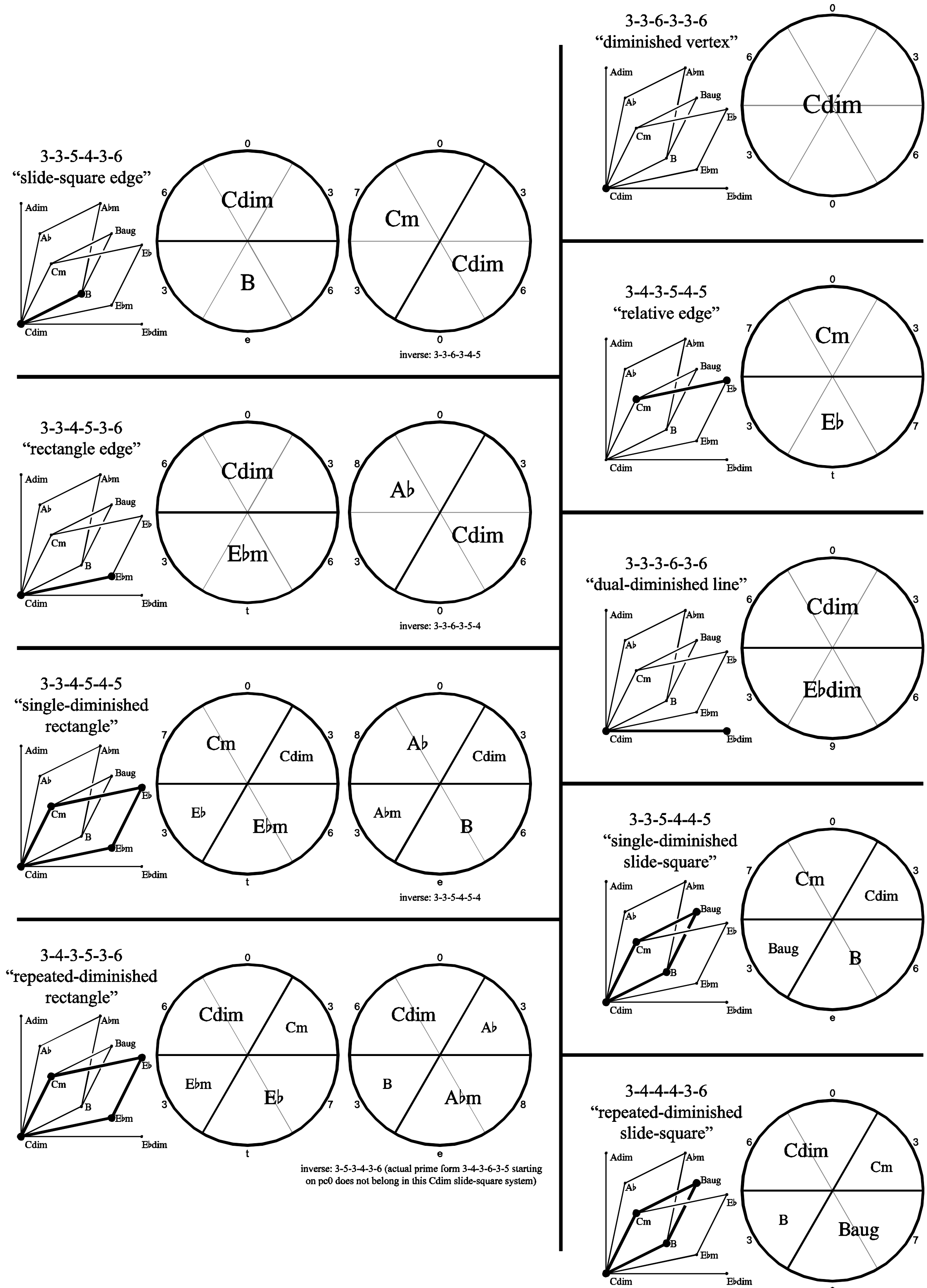

Figure 2: Nine permutations within the "billfold" extension

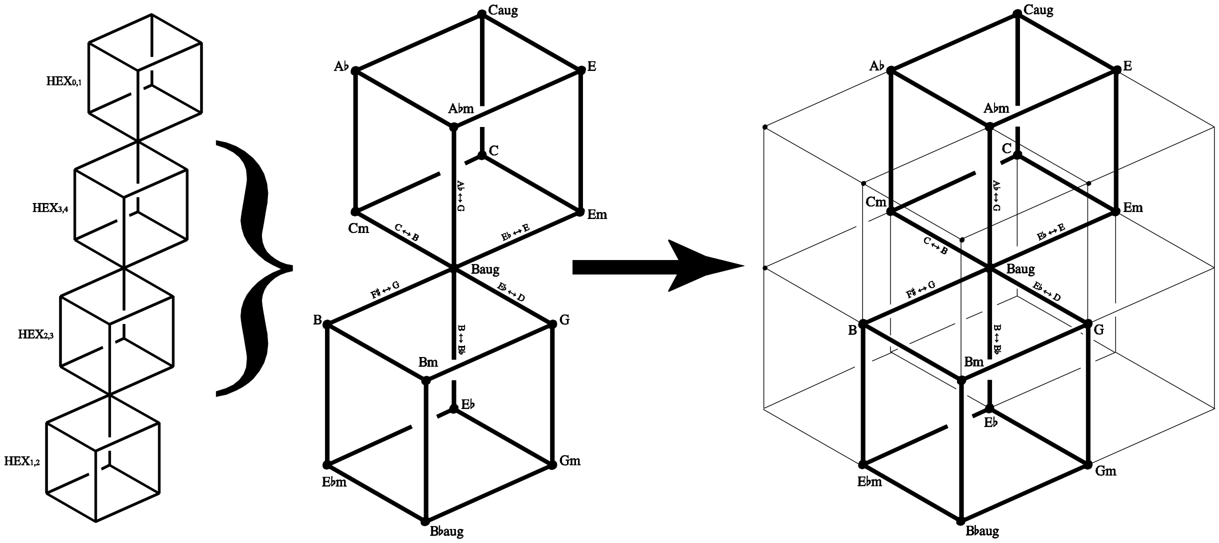

Figure 3: The Cube Dance superimposed over a 2x2x2 cube

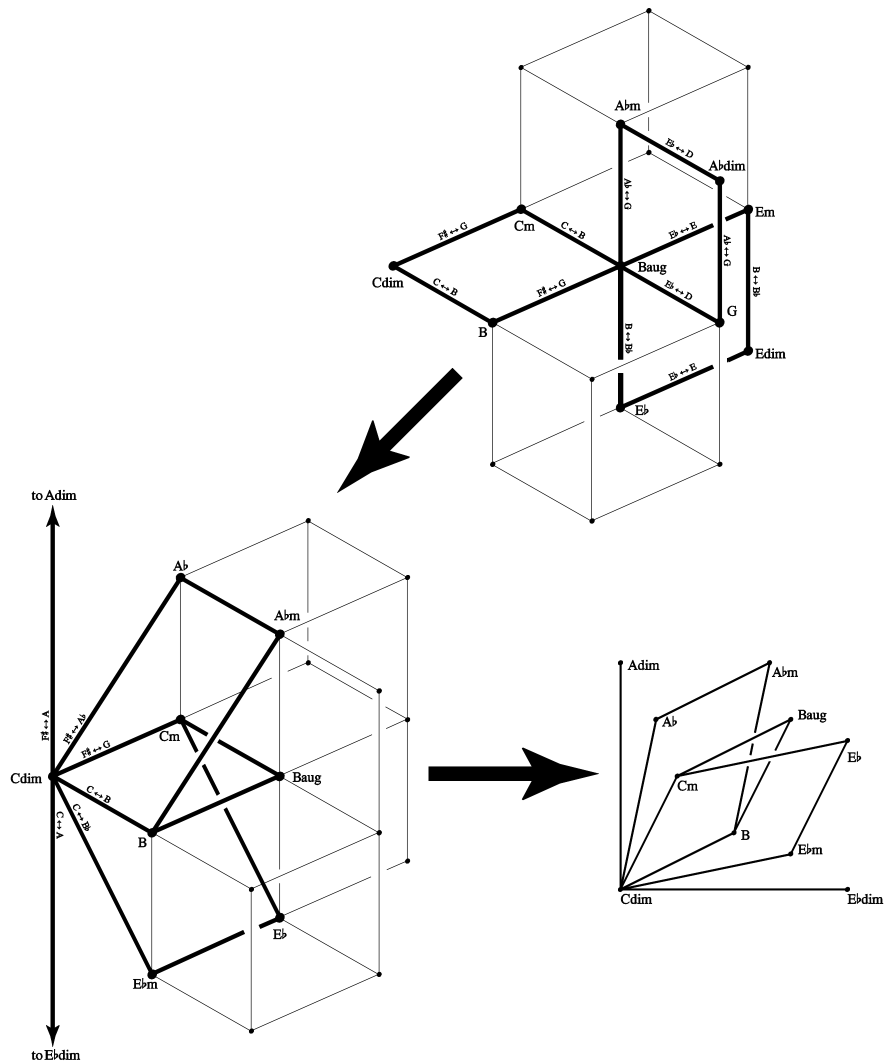

Figure 4: The "billfold" model as an extension of the Cube Dance

permutations may then be represented within this "billfold" in the same way that the original nine permutations may be represented within a hexatonic cube. The "relative-edge" permutation may be represented in either "billfold" of the two diminished triads to which it is closest.

I have labeled some of the permutations in Figure 2 with the term "slide-square" to distinguish them from those permutations whose square shape originates from the hexatonic cube. The "slide-square" is, of course, the square at the center of the "billfold" with a diminished triad and augmented triad at opposite corners. The two consonant triads at the other corners, B-maj and C-min, are related by David Lewin's Slide operation. The surfaces containing whole tone voice-leading paths, on the other hand, are simply labeled "rectangle," as their unique shape requires no special designation.

II. Scale and Key Signature Charts

Each of these eighteen permutations can be treated as a single snapshot in time that, when ordered and arranged in a sequence, may be used as a compositional design for a musical piece to be heard in the hexagonal virtual room. While the innumerable possibilities might seem too daunting to contemplate, a simple way to begin would be to create a sequence of single-triad rooms (i.e., those I have labeled "vertex" in Figures 1 and 2) following conventional harmonic forms. Rooms divided between two or more triads ("edges," "squares," etc.) may then be substituted in wherever the option is both available and desired. One such example is shown in Figure 5, which presents a simple compositional design that can be directly interpreted as akin to an exercise in first-species counterpoint. Upon initial glance, then, the process of fleshing it out into a more respectable musical piece through melodic gestures and ornamentation might seem rather straightforward.

A complication immediately arises, however, once we consider the key of this piece. Since it is in both C-maj and C-min, how does one create melodies to be sounded by those voices that may be heard in either key?[7] Since we arrived at a means of creating harmonies for the virtual room by way of the Cube Dance, it seems intuitive to seek out another geometric model, this time involving voice-leading between scales, to help us determine the rules for creating melodies. A section of one such model, Dmitri Tymoczko's scale lattice for seven-note Pressing scales,[8] is shown in Figure 6.[9] Like the two adjacent hexatonic cubes from the Cube Dance, these four adjacent cubes of the scale lattice may be viewed as part of a larger 2x2x2 cube to yield certain insights.

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[7] I am assuming a completely tonal context for this piece; understandably, any attempt to stretch the boundaries of tonality at this stage would be premature.

[8] Tymoczko, "Scale Networks and Debussy": 239-40.

[9] Structurally, this is the same model as that found in Tymoczko's paper. I only changed its orientation and designated the center vertex as C-diatonic.

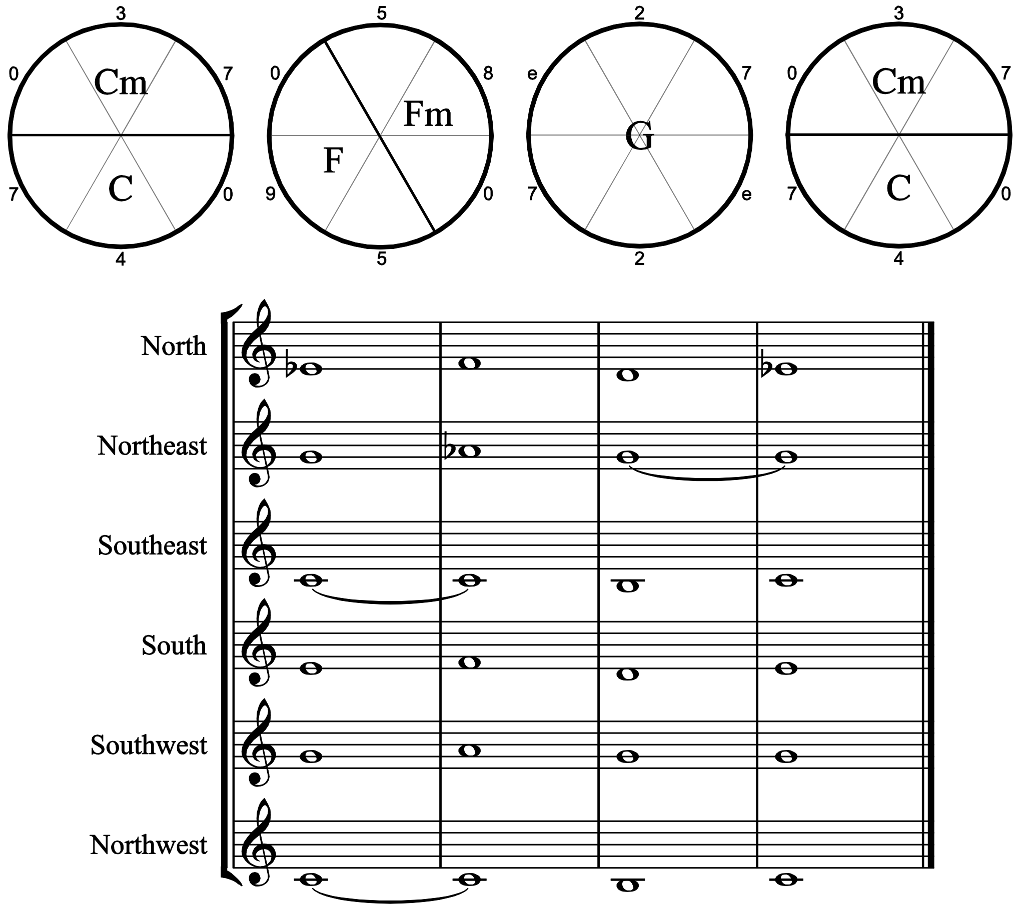

Figure 5: A very simple compositional design

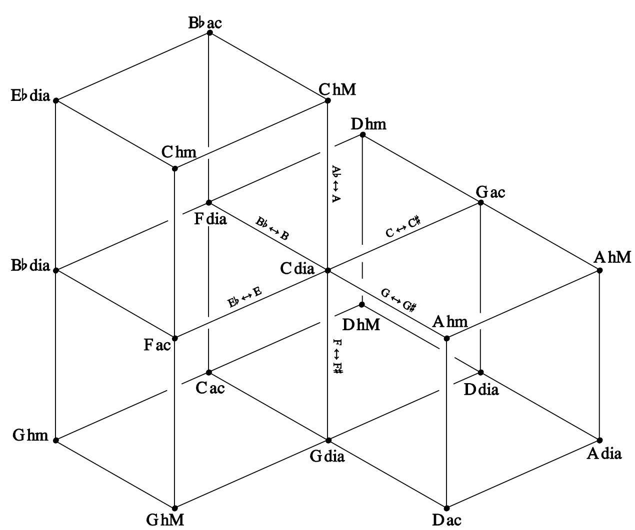

Figure 6: A portion of Dmitri Tymoczko's scale lattice

Diatonic, major and minor		
G♭	E♭m	C♭ G♭ D♭ A♭ E♭ B♭
D♭	B♭m	G♭ D♭ A♭ E♭ B♭
A♭	Fm	D♭ A♭ E♭ B♭
E♭	Cm	A♭ E♭ B♭
B♭	Gm	E♭ B♭
F	Dm	B♭
C	Am	---
G	Em	F♯
D	Bm	F♯ C♯
A	F♯m	F♯ C♯ G♯
E	C♯m	F♯ C♯ G♯ D♯
B	G♯m	F♯ C♯ G♯ D♯ A♯
F♯	D♯m	F♯ C♯ G♯ D♯ A♯ E♯

Acoustic (major with raised 4th and lowered 7th)	
G♭	F♭ --- G♭ D♭ A♭ E♭ B♭
D♭	C♭ --- D♭ A♭ E♭ B♭
A♭	G♭ --- A♭ E♭ B♭
E♭	D♭ --- E♭ B♭
B♭	A♭ --- B♭
F	E♭ ---
C	B♭ --- F♯
G	--- C♯
D	F♯ --- G♯
A	F♯ C♯ --- D♯
E	F♯ C♯ G♯ --- A♯
B	F♯ C♯ G♯ D♯ --- E♯
F♯	F♯ C♯ G♯ D♯ A♯ --- B♯

Harmonic major (major with lowered 6th)	
G♭	E𝄫 --- --- C♭ G♭ D♭ A♭ E♭ B♭
D♭	B𝄫 --- --- G♭ D♭ A♭ E♭ B♭
A♭	F♭ --- --- D♭ A♭ E♭ B♭
E♭	C♭ --- --- A♭ E♭ B♭
B♭	G♭ --- --- E♭ B♭
F	D♭ --- --- B♭
C	A♭ --- ---
G	E♭ --- --- F♯
D	B♭ --- F♯ C♯
A	--- C♯ G♯
E	F♯ --- G♯ D♯
B	F♯ C♯ --- D♯ A♯
F♯	F♯ C♯ G♯ --- A♯ E♯

Harmonic minor (minor with raised 7th)	
E♭m	C♭ G♭ --- A♭ E♭ B♭
B♭m	G♭ D♭ --- E♭ B♭
Fm	D♭ A♭ --- B♭
Cm	A♭ E♭ ---
Gm	E♭ B♭ --- F♯
Dm	B♭ --- --- C♯
Am	--- --- G♯
Em	F♯ --- --- D♯
Bm	F♯ C♯ --- --- A♯
F♯m	F♯ C♯ G♯ --- --- E♯
C♯m	F♯ C♯ G♯ D♯ --- --- B♯
G♯m	F♯ C♯ G♯ D♯ A♯ --- --- F𝄪
D♯m	F♯ C♯ G♯ D♯ A♯ E♯ --- --- C𝄪

Figure 7: Key signatures for all diatonic, acoustic, and harmonic major and minor scales

The six voice-leading paths leading away from C-diatonic as the center vertex are F to F♯, C to C♯, G to G♯, B to B♭, E to E♭, and A to A♭. These correspond to the first three accidentals found in the key signatures on either side of C-maj along the circle of fifths; this is not a coincidence, for as Tymoczko and Julian Hook have pointed out, key signatures translate directly into voice-leading paths.[10] While each succeeding diatonic scale along the circle of fifths must add or remove an accidental in a specific order, however, the scale lattice within the 2x2x2 cube shows all the tonally functional seven-note scales which may be created through any combination of its six voice-leading paths. (Not surprisingly, the vertices of the 2x2x2 cube which are absent in the scale lattice represent impossible voice-leading combinations: G♯ and A♭ are enharmonic notes, while A♭ with C♯ and E♭ with G♯ produce consecutive intervals spanning a perfect fourth.)[11]

Key signatures are thus a convenient way to signify in the compositional design which melodic notes are available for each given voice. I must specify here that such keys and key signatures are always local; they apply only to one particular region of the virtual room, and only for the indicated duration of time. Figure 7 shows a chart of key signatures for all the diatonic, acoustic, and harmonic major and minor scales within six scales away from C-maj or A-min along the circle of fifths. Each scale deviates from its diatonic counterpart by raising or lowering the appropriate scale degrees. For example, to obtain the key signature of G-acoustic, we begin with a single F♯, the key signature of G-diatonic. We then raise the fourth from C to C♯ and lower the seventh from F♯ to F, giving us a single C♯ as the result.

For the sake of musical continuity, it is probably desirable as a general rule for each snapshot in time to contain as few keys as possible, with minimal differences between those keys. Assigning a key signature to each region of the virtual room simply involves ensuring that the notes of the triad heard within that room do not clash with those in the scale signified by that key signature. The process is naturally straightforward for a single-triad "vertex" room, while the double-triad "edge" room also presents a relatively simple case, for there is only one voice-leading path to consider. In fact, the two respective triads of the "relative edge" or "leading-tone edge" may easily share the same key signature.

Of course, different key signatures are required for the "parallel edge," such as the one divided between C-maj and C-min found in Figure 5. Because the voice-leading path between these two triads is E to E♭, we simply need to find two scales that only differ in this one respect and do not conflict with the shared tones of C and G. Possible options include B♭-acoustic (B♭,A♭) with E♭-diatonic (B♭,E♭,A♭), and C-acoustic (B♭,F♯) with G-harmonic minor (B♭,E♭,F♯), as shown in Figure 8. (The bold, gray line extending slightly beyond the circle represents the boundary between the two keys.) A very important point to mention is that key signatures apply to wedges, not voices. That is to say, individual voices are constrained, not enabled, by the

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[10] Tymoczko, "Voice Leadings as Generalized Key Signatures," 2005.
[11] Dmitri Tymoczko, "The Consecutive-Semitone Constraint on Scalar Structure: A Link Between Impressionism and Jazz," *Intégral* 11 (1997): 135-146. Consecutive intervals spanning a perfect fourth constitute a subset of some larger scale, failing the criteria for a tonally functional scale.

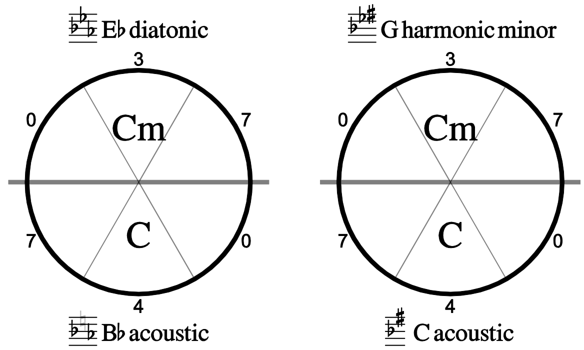

Figure 8: Some possible local key signatures for the "parallel edge" room

designated key signatures for all three wedges in which they may be heard. A glance at Figure 8 makes this point clear: because the two western and two eastern voices are heard in both C-maj and C-min, all four of these voices are unable to play either E or E♭ in their melodies. Thus, the difference between the two key signatures is heard solely in the northern and southern voices.

The task of allocating key signatures among the different regions of the hexagonal room becomes more difficult as the number of different triads increases. To simplify the process, it is useful to compile a chart showing all the scales that fully contain any particular triad, as in Figure 9. Four tables are shown, representing major, minor, augmented, and diminished triads, respectively. Thus, for example, the C-maj triad is fully contained in the following scales: C-diatonic, F-diatonic, G-diatonic, C-harmonic major, F-harmonic major, C-acoustic, B♭-acoustic, E-harmonic minor, and F-harmonic minor. These nine scales, in turn, may be plotted on the scale lattice, as shown in bold, black lines in Figure 10. It is easy to see here that all the scales which fully contain the C-maj triad constitute a plane centered on C-diatonic that extends outward to the limits imposed both by permissible voice-leading and by the helical structure of the scale lattice, forming a "staircase" shape. Similarly, those scales which contain the C-min triad, shown in bold, gray lines, form an inverse "staircase" shape. Figure 10 thus shows us that any two scales separated only by the voice-leading path between E and E♭ may be used for the respective melodies heard by each triad in the permutation divided between C-maj and C-min. (Of course, some scales will carry greater musical weight than others.)

The same process may be applied to those permutations which divide the room between four different triads, such as "squares" and "rectangles." First, one groups together all the scales which fully contain each triad, such as the "staircase" shape for consonant triads; then, one looks for instances in which the intersection of these shapes forms a closed path on the scale lattice. (In the case of a permutation with four triads, a closed path can only be a single voice-leading path between two vertices, or two perpendicular voice-leading paths that form a square.) Because "square" and "rectangle" permutations always include augmented and/or diminished triads, this process is not as difficult as it might appear, for the shapes formed by the scales which fully contain augmented and diminished triads occupy different planes, creating more opportunities for the desired voice-leading paths to intersect around one location on the scale lattice. In a manner similar to the relationship between the arrangement of triads in a virtual room and the Cube Dance, then, the distribution of local key signatures in a virtual room will always be isomorphic to a closed path within the scale lattice.

Maj	dia	ac	hM	hm
G♭	G♭ C♭ D♭	F♭ G♭	G♭ C♭	B♭m C♭m
D♭	D♭ G♭ A♭	C♭ D♭	D♭ G♭	Fm G♭m
A♭	A♭ D♭ E♭	G♭ A♭	A♭ D♭	Cm D♭m
E♭	E♭ A♭ B♭	D♭ E♭	E♭ A♭	Gm A♭m
B♭	B♭ E♭ F	A♭ B♭	B♭ E♭	Dm E♭m
F	F B♭ C	E♭ F	F B♭	Am B♭m
C	C F G	B♭ C	C F	Em Fm
G	G C D	F G	G C	Bm Cm
D	D G A	C D	D G	F♯m Gm
A	A D E	G A	A D	C♯m Dm
E	E A B	D E	E A	G♯m Am
B	B E F♯	A B	B E	D♯m Em
F♯	F♯ B C♯	E F♯	F♯ B	A♯m Bm

Min	dia	ac	hM	hm
E♭m	G♭ C♭ D♭	G♭ A♭	B♭ C♭	B♭m E♭m
B♭m	D♭ G♭ A♭	D♭ E♭	F G♭	Fm B♭m
Fm	A♭ D♭ E♭	A♭ B♭	C D♭	Cm Fm
Cm	E♭ A♭ B♭	E♭ F	G A♭	Gm Cm
Gm	B♭ E♭ F	B♭ C	D E♭	Dm Gm
Dm	F B♭ C	F G	A B♭	Am Dm
Am	C F G	C D	E F	Em Am
Em	G C D	G A	B C	Bm Em
Bm	D G A	D E	F♯ G	F♯m Bm
F♯m	A D E	A B	C♯ D	C♯m F♯m
C♯m	E A B	E F♯	G♯ A	G♯m C♯m
G♯m	B E F♯	B C♯	D♯ E	D♯m G♯m
D♯m	F♯ B C♯	F♯ G♯	A♯ B	A♯m D♯m

Aug	ac	HM	hm
G♭ B♭ D F♯aug	F♭ A♭ C E	G♭ B♭ D F♯	E♭m Gm Bm D♯m
D♭ F A C♯aug	C♭ E♭ G B	D♭ F A C♯	B♭m Dm F♯m A♯m
A♭ C E G♯aug	G♭ B♭ D F♯	A♭ C E G♯	Fm Am C♯m E♯m
E♭ G B D♯aug	D♭ F A C♯	E♭ G B D♯	Cm Em G♯m B♯m

Dim	dia	ac	HM	hm
Fo	G♭	C♭ D♭	E♭ G♭	E♭m G♭m
Co	D♭	G♭ A♭	B♭ D♭	B♭m D♭m
Go	A♭	D♭ E♭	F A♭	Fm A♭m
Do	E♭	A♭ B♭	C E♭	Cm E♭m
Ao	B♭	E♭ F	G B♭	Gm B♭m
Eo	F	B♭ C	D F	Dm Fm
Bo	C	F G	A C	Am Cm
F♯o	G	C D	E G	Em Gm
C♯o	D	G A	B D	Bm Dm
G♯o	A	D E	F♯ A	F♯m Am
D♯o	E	A B	C♯ E	C♯m Em
A♯o	B	E F♯	G♯ B	G♯m Bm
E♯o	F♯	B C♯	D♯ F♯	D♯m F♯m

Figure 9: Permissible scales for all major, minor, augmented, and diminished triads

A final point to make is that it is impossible to find compatible key signatures for all the different triads in "hexagon" permutations. This seems to be a byproduct of the fact that the underlying structure of the Cube Dance is based on equal division of the octave by three (a major third), while that of the scale lattice is based on equal division of the octave by four (a minor third).

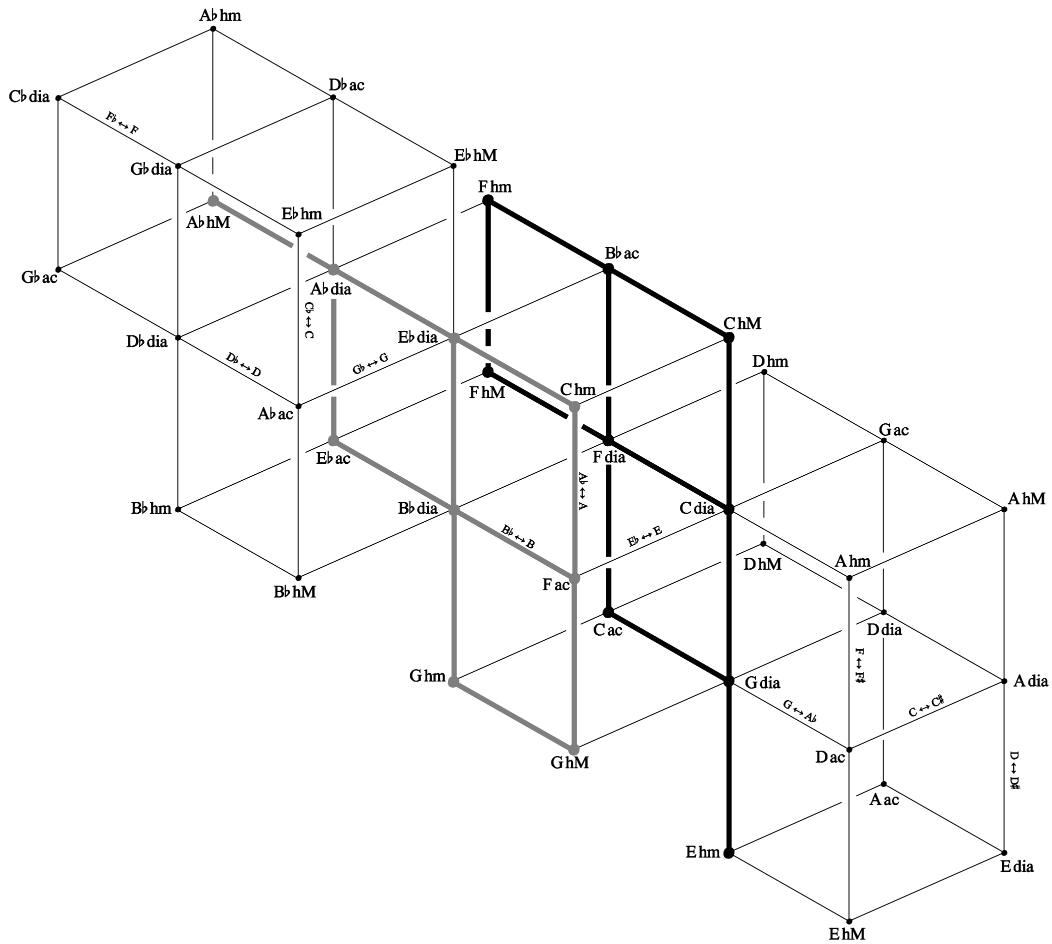

Figure 10: Permissible scales superimposed over the scale lattice

III. Creating and Realizing a Compositional Design

Figure 11 shows the compositional design that I used to create my musical example, "Amnestic Hexagon." The manner by which I notated the pitch-classes sounded by each voice, the triads heard within each wedge, and the key signatures denoting the melodic scales permitted within each region have all been explained in this paper. One point I wish to reiterate: a key signature limits the notes which may be played by any particular voice. For example, in the fourth snapshot of rehearsal letter A in Figure 11, the melody in the northwestern voice must be compatible with three scales: G-diatonic, E-harmonic minor, and E-harmonic major. As such, it cannot contain any of the following notes: D, D♯, G or G♯.

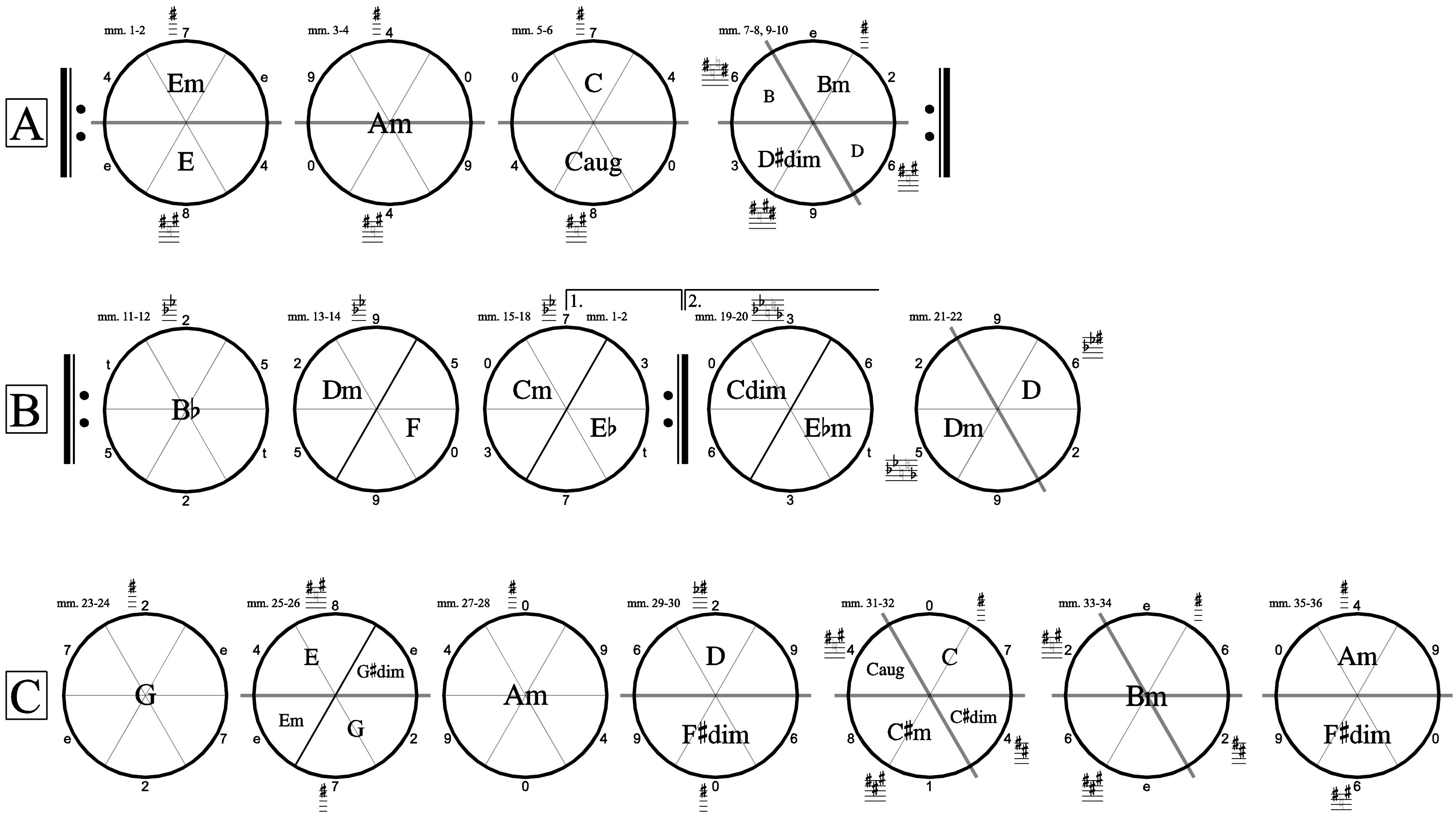

Figure 11: The compositional design for "Amnestic Hexagon"

The chord progressions and modulations chosen for this compositional design largely conform to the idioms of pop and rock music. As it turns out, the process of adding permutations containing two or more different triads was very similar to that of choosing between comparably appropriate chords to fill a particular bar in a conventional piece. The reason for this should be obvious: in both cases, the different triads being considered are often related by voice-leading. This manner of virtual composition thus works like the musical equivalent of the popular "Choose Your Own Adventure" series of children's books. In each story of this genre, the fate of the protagonist is entrusted to the reader, who is asked to choose between different scenarios, then directed to the appropriate page number to witness the outcome of his or her decision. Likewise, a listener in the hexagonal virtual room has the ongoing option to hear greater modal mixture, a different instrumental arrangement, and so forth.

A thorough comparison between the compositional design and the final score of "Amnestic Hexagon" will reveal many glaring inconsistencies, such as a melodic phrase being played by an instrument which does not match the pitch class it was assigned during that particular section of the piece. This was unavoidable, for my reasoning in each such case was to privilege aesthetic considerations over academic discourse. As such, "Amnestic Hexagon" by no means represents an ideal defense of the methods that I propose in this paper; rather, my goal was to present a realistic example of just how virtual composition might be undertaken by a composer driven by no other motive than to compose music for its own sake. The construction of any compositional design and its realization into a final piece necessarily requires making some artistic compromises, and this method of virtual composition that I propose is no different in that respect.

Still, it is legitimate to ask whether the problems I encountered during the creation of this piece are an intrinsic part of the system itself. One such problem, for example, is that the location of the bass instrument is always fixed while the location of the root note differs for each triad, leading to the constant presence of first and second inversion chords where they are not always desired. My desire to find a solution to this problem and others like it will serve as the impetus for a future paper. In the meantime, it is my belief that the discovery of methods for avoiding or conquering these problems will arise most naturally and effectively through the continued application of this method of virtual composition.

IV. Seventh Chords in the Virtual Room

While seventh chords would greatly expand the musical palette of the virtual room, any attempt to accommodate chords of different cardinalities in this environment will lead to complications, reflecting the inherent difficulties of representing such chords within a single geometric model.[12]

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[12] Dmitri Tymoczko, "Geometrical Methods in Recent Music Theory," *Music Theory Online* 16, no. 1 (March 2010): 16.1.7.

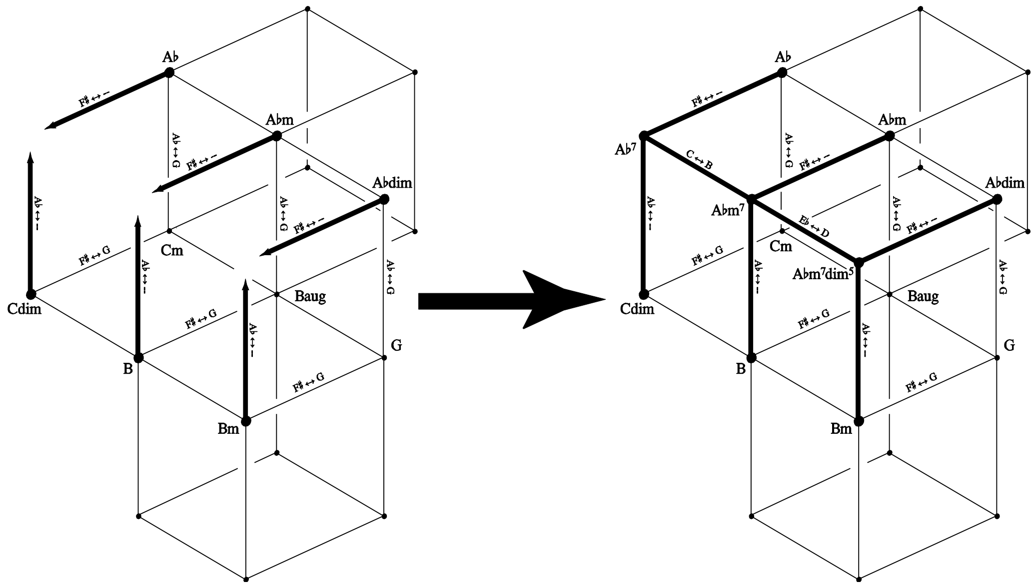

Figure 12: Split/fuse transformations in the Cube Dance

I will now describe two possibilities for the inclusion of seventh chords, both in terms of virtual room arrangement and their representation within a geometric model, along with the problems associated with each one.

In Figure 3, the vertices of the 2x2x2 cube which are absent in the two hexatonic cubes superimposed above it represent impossible voice-leading combinations for triads. For example, three vertices would require G to move to both F♯ and A♭ simultaneously. However, an alternative interpretation is possible: the split/fuse transformation described by Callender.[13] In this case, the vertices at the opposite end of the voice-leading paths from G to F♯ and G to A♭ represent seventh chords in which the G has split into both F♯ and A♭. This is shown in Figure 12, which also neatly demonstrates the voice-leading relations between minor sevenths, dominant sevenths, and half-diminished sevenths of the same root pointed out by Douthett and Steinbach in their Power Towers.[14]

A straightforward means of arranging voices in a virtual room isomorphic to a closed path that includes a split/fuse transformation is to allow one voice to represent two pitch classes, as shown in Figure 13. This solution necessarily renders seventh chords subordinate to triads, reflecting the general reality of tonal music. The drawback, of course, is that it precludes the possibility of expanding into a post-tonal vocabulary that treats chords of greater cardinality as distinct entities, independent of the triads they subsume.

A solution that removes seventh chords from the context of triads is the “octagonal” virtual room, as shown in Figure 14. While such an arrangement of voices may be represented by

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[13] Callender, 224-32. Callender uses the split/fuse transformation to describe relations between specific collections of 6, 7, and 8 cardinality.
[14] Douthett and Steinbach, 255-256.

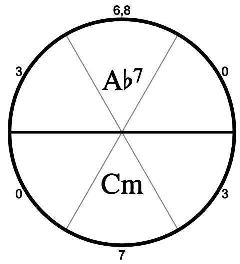

Figure 13: A split/fuse transformation in the hexagonal room

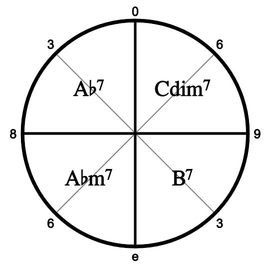

Figure 14: An octagonal virtual room

a geometric model that treats all vertices as seventh chords, such as the Power Towers, it is perhaps preferable from a musical standpoint to envision the corresponding geometrical model as one which includes triads. Figure 15 shows that the voice-leading paths already present within the Cube Dance may be extended to display the relations between seventh chords found in the Power Towers along a planar surface. Such a representation is far from ideal, of course, and belies the multidimensional character of its voice-leading relations. For example, A♭-dom7 is connected to two paths that transform A♭ to A: a horizontal one leading to A-dim7 and a vertical one leading to C-dim7, ignoring the fact that A-dim7 and C-dim7 are the same chord in geometric space. Of more immediate concern is how the octagonal room may be reconciled with the hexagonal room. While each may alternate in succession, how are voices reoriented when a split/fuse transformation takes place? Is it possible for hexagonal and octagonal wedges to coexist during a single snapshot in time?

Other possible means for including seventh chords may be considered, such as dividing the virtual room into a greater number of wedges, or reducing the physical range of sound propagation for each individual voice to less than a semicircle. However, these suggestions similarly neglect to address the fundamental problems that are raised when chords of different cardinalities are combined in the virtual room: How are voices added, removed, or reoriented in a seamless manner befitting a fluid and cohesive musical piece? It is quite possible that the most satisfactory solution will include mechanical components in addition to theoretical ones, taking advantage of the options afforded by digital audio recording and software programming. I will explore such possibilities in a future paper.

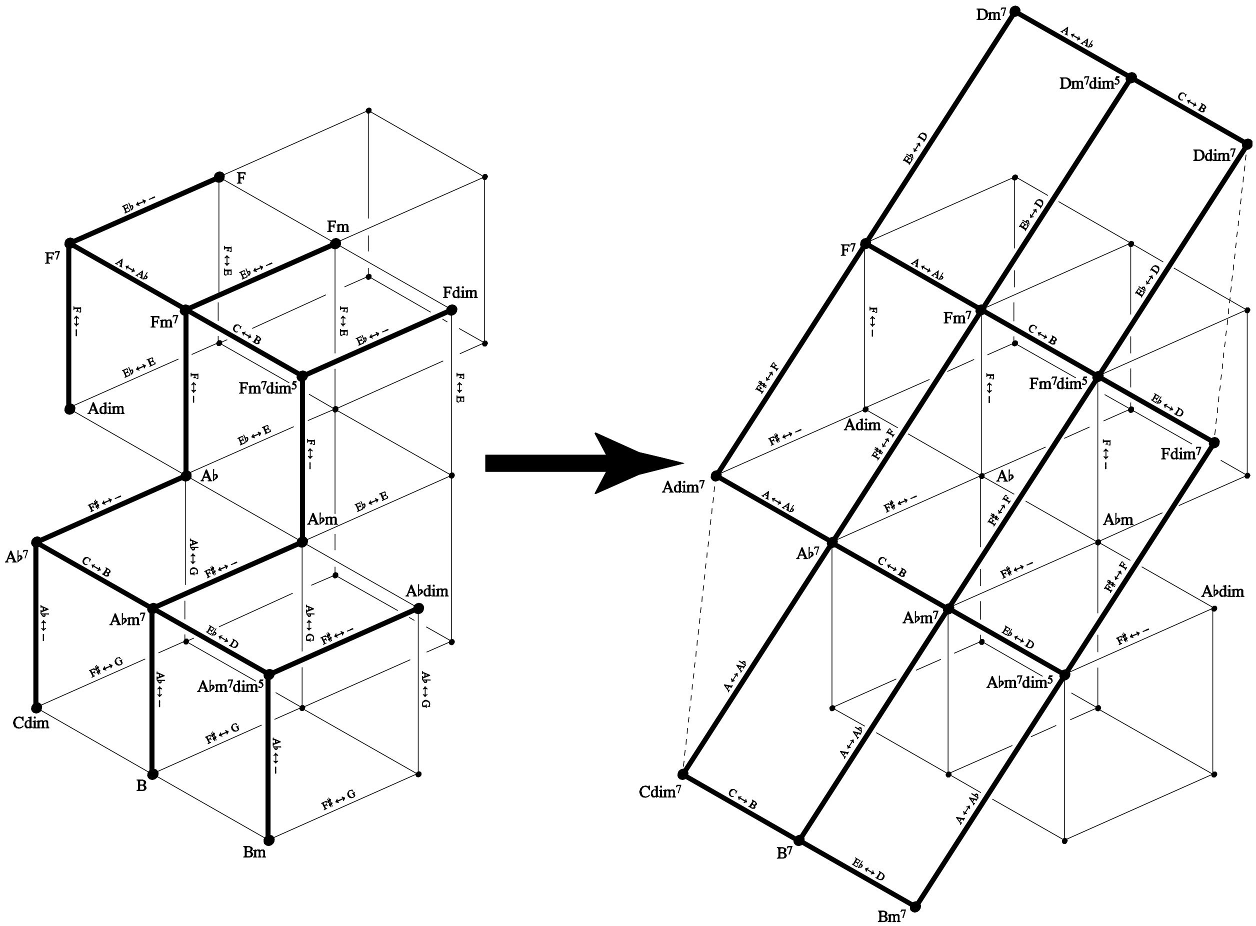

Figure 15: Power Towers as an extension of the Cube Dance

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