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Serialism in Stockhausen's *Formel*

Karlheinz Stockhausen composed *Formel* towards the end of 1951 as a natural continuation of the pointillism developed in his previous piece *Kreuzspiel*, with entire melodic and harmonic complexes now being systematically arranged in place of single notes.[1] While the original intent was to have it be the first movement of *Spiel*, this idea was abandoned due to its difference in structural organization with the other two movements. *Formel* did not receive its first performance until 1971, after Stockhausen realized that certain ideas for thematic development explored during the composition of *Mantra* had been latent all along in the former piece.[2]

Previous analytical research on *Formel* is rather sparse, and perhaps not without reason: beyond the serialist procedure underlying the formal structure of the piece, there is not much thematic or harmonic development that merits lengthy explanation. Nevertheless, because of its initial dismissal by Stockhausen as a detour in his artistic growth, *Formel* presents a unique opportunity for us to examine how the various creative techniques he employed throughout his life might have been informed by a commonality of thoughts and objectives, even if unbeknownst to the composer himself at the time. In this paper, I will analyze the serialist techniques underlying the composition of *Formel*, then place the piece within the context of Stockhausen's early oeuvre to trace the development of his compositional approach.

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[1] Michael Kurtz, *Stockhausen: A Biography* (London: Faber and Faber, 1992), 43.
[2] Robin Maconie, *The Works of Karlheinz Stockhausen* (London: Oxford University Press, 1976), 29.

The two sides of both front and back cover in Universal Edition's 1974 printing of *Formel* reproduce the first four pages of the score with Stockhausen's own handwritten marks and various notes superimposed in bright colors. From these clues alone, several details may be quickly gleaned. First, the entire piece is made up of twelve distinct "Glieder," or members, which are segments of music consisting of one to three bars in length, each invariant in its specific values for pitch classes, note durations, and articulations (but not register or instrumentation).

Second, the first four pages of the score show the twelve Glieder arranged in their proper sequence, or what Robin Maconie refers to as the "initial gestalt," which introduces the source material that is to be permutated and rearranged throughout the rest of the piece.[3] During this initial gestalt, the vibraphone plays each melodic segment as a single, uninterrupted gesture, as shown in Figure 1.[4] While each Glied is unique in the character of its melodic articulations—the tremolos that finish up no. 4, for example, or the acciaccature that make up no. 9—the similarity of interval content, proximity in register, and slurred notes across adjacent Glieder help to establish the gestalt as a single, unified entity. Moreover, the first six Glieder smoothly increase in dynamics from pianissimo to fortissimo, while the next six take the exact same course in reverse.

The third and final point is that the note durations and time signatures of each Glied follow a simple pattern. In no. 1, the melody contains only pc0 and is

[3] Maconie, 27.

[4] Due to the limitations of my notation software, two of the time signatures shown in Figure 1 are not completely faithful to the score. They are the second and third bars of Glied no. 7, which Stockhausen allows to be divisible into dotted-note beats. For example, the second bar of Glied no. 7 is written in the score as (1/𝅗𝅥. + 2/𝅘𝅥𝅯.). I notate it as the temporally equivalent (3/𝅗𝅥).

defined by a single dotted-half-note duration. In Glied no. 2, the melody consists of two pitch classes and may be roughly divided into two durations, each lasting for eleven sixteenth notes. Meanwhile, the harmonic rhythm in no. 1 may be divided into twelve sixteenth notes, while that of no. 2 may be heard as eleven eighth notes. Finally, Glied no. 12 contains all twelve pitch classes, each a sixteenth-note in duration.[5] While the harmony in Glied no. 1 contains all eleven pitch classes not found in the overlying melody, this logic is not repeated in no. 2 or in any of the other Glieder.

In other words, while the melody in each subsequent Glied includes one additional pitch class, each of which lasts for the duration of one fewer sixteenth note, the harmony in each subsequent Glied contains one fewer pulse, each of which lasts the duration of an additional sixteenth note. This inverse relationship is shown in Figure 2. The time signatures given merely reflect the sum of note durations contained within each Glied, and seem to vary throughout the piece based on performance expectations. For example, Glied no. 10 at bar 22 is indicated by three bars of the following time signatures: four quarter notes, two quarter notes, and three eighth notes, respectively. At bar 191, however, it is indicated by a bar of five quarter notes, followed by one of five eighth notes.

After the introductory presentation of the twelve Glieder as a complete sequence, the remainder of the piece rearranges seven additional sequences in the

[5] In the initial gestalt, the vibraphone begins the lead melody of Glied no. 12 with a sixteenth-note rest rather than the expected pc2, which is heard in the celesta. Throughout the remainder of the piece, the instrument assigned the lead melody of Glied no. 12 either sounds pc2 as the first note or else has it covered by another instrument. For the purposes of my analysis, I consider the melody of Glied no. 12 to include all twelve pitch classes, with pc2 as its first note.

manner shown in Figure 3, which depicts the piece's overall structure. The horizontal row at the top represents the first twelve Glieder. Below it, each sequence is rotated vertically to form a column, with each subsequent column offset from the one to its left by one row. The piece then unfolds by starting at the top, proceeding left to right through each row, all the way to the bottom. Glied no. 1 is then repeated after one bar of rest to end the piece.

There are two features of this arrangement that serve to shuffle the Glieder from their original order. The first is the offsetting of each column to the right, which makes each row an exact reverse segment of the original gestalt. This ordering does not change, however: Glied no. 3 always follows no. 4 in every single row in which they both appear. The second feature is the manner in which the changing cardinality of each row interacts with Stockhausen's choice of seven columns. Under this arrangement, Glied no. 1 gets followed by nos. 2 through 8 in each of its appearances, no. 12 gets preceded by nos. 5 through 11, and nos. 2 through 4 each precede the seventh Glied behind it in the sequence.

Figure 4 shows a visual map of *Formel* in its entirety, with time measured in bar numbers along the x-axis and instruments listed along the y-axis.[6] For each instrument, each complete Glied in which it plays the lead melody is shaded in dark gray, and each individual bar in which it plays a harmony or countermelody is shaded in light gray. Vertical lines demarcate the rows of Glieder identified in Figure 3. Horizontal lines separate the orchestra into four choirs: woodwind, vibraphone

[6] While the color code I use to distinguish the different Glieder in this graph is not an essential component, I believe it illustrates more effectively how the Glieder are distributed throughout the piece.

and glockenspiel, keyboards and harp, and strings. The rationale for this instrumental division shall be explained later.

It can be seen in this graph that the vibraphone plays the central role in *Formel*. Beyond simply introducing the initial gestalt, it ties the piece together through each of its reappearances. The vibraphone is restricted almost exclusively to repeating exact segments of the melody it played in the initial gestalt, staying within the same register and using the same articulations. The two exceptions are in bars 41 and 91, where it lingers to sound out minimal harmonic gestures after it has performed the melodic segment.

The vibraphone's reappearances follow a systematic pattern: it plays the first Glied that introduces each new row in the overall structure. Figure 4 shows this pattern clearly, as it is the first instrument to have the lead melody after each vertical line. In performance, this lends the impression of the vibraphone gradually entrusting the melody to the other instruments, contenting itself to reappear only upon every seventh Glied before disappearing altogether.

An interesting point of observation is that when an instrument other than the vibraphone is playing a melody at any given time after the fourth row following the initial gestalt, this instrument will either play the melody for two consecutive Glieder, or will accompany another instrument that does so. For this reason, the consistently reverse ordering of the Glieder within each row is less likely to grow tiresome. The juxtaposition of the melodies of Glieder nos. 5 and 4 in one row might be explored by the same instrument, for example, while the next row will juxtapose nos. 6 and 5, with nos. 5 and 4 now broken up by different timbres.

Formel was composed in 1951, several years before Stockhausen began his work in the electronic studios, and thus does not reflect the insight he acquired from that experience. Still, its composition illustrates the similarity of thought and procedure in both serialism and electronic music, and might help explain Stockhausen's success as a composer in each realm. Figure 4 shares many features in common with the visual maps of Stockhausen's electronic pieces, namely attentiveness to both density and spatial distribution.

The division of the instruments into four choirs should now be evident. The woodwinds appear during the initial gestalt only to double the vibraphone's melody; afterwards, their presence is concentrated towards the front of the piece, as is that of the vibraphone and glockenspiel. By contrast, the other two choirs make their heaviest contributions towards the end. This makes sense given the general order in which each choir is given the melody for each row after the initial gestalt. In each case the vibraphone is immediately followed by the woodwinds; then, as permitted by the expansion of each row, the woodwinds are followed by the bowed strings, and then by the choir of struck and plucked instruments. Conversely, as the rows begin to collapse in size towards the end of the piece, the vibraphone is the first to drop out, followed by the woodwinds, and then by the bowed strings.

The recurrence of the melody in the vibraphone at the beginning of each row may be viewed as a complete restatement of the initial gestalt broken up and interspersed with new melodic material, much like segments of analog tape taken from different reels might be cut and spliced together to create a new track. This analogy proves apt when viewing the graph in Figure 5, which shows how many

instances a particular instrument plays the lead melody of each Glied for the entire piece. While there are several exceptions, the density stays relatively uniform.

Except for the double bass, when a Glied is assigned multiple times to the same instrument, the additional instance always occurs in either the initial gestalt or in the final appearance of Glied no. 1 at the end of the piece. In each situation, every instrument other than the vibraphone plays the melody in a different register; on the other hand, the vibraphone always plays the same melody in the same register, with exactly the same dynamics and articulations. It seems likely, then, that Stockhausen consciously treated each complete sequence of the gestalt as a limited and exhaustible supply, much like segments of analog tape would be. Thus, serialism served the interests of his earlier pieces in the same way that technology of the age would later serve his electronic pieces, by imposing natural constraints that conserve and distribute raw material in a manner maximizing listener interest.

The influence of Arnold Schoenberg's twelve-tone technique on Stockhausen is readily visible in *Formel*. The left table of Figure 6 shows the pitch-class constituency for the melody of each Glied. Beginning with one pitch class in Glied no. 1, each subsequent Glied contains an additional member until all twelve pitch classes are represented in Glied no. 12. No pitch class is repeated between adjacent Glieder until no. 7, which misses the chance to fill in a pc2 left out by Glied no. 6, instead repeating pcs 3 and 5. If Stockhausen's goal was to create uniform density, then this is a curious anomaly in light of the relative deficiency of pc2s overall. No other Glied from that point misses an opportunity to fill in a pitch class absent from the one before.

The right table of Figure 6 shows the order of the pitch classes when the Glieder are played in proper sequence during the initial gestalt. Three complete sequences of all twelve pitch classes may be readily discerned. The first is, of course, Glied no. 12, whose melody [2569t1e87430] begins with an ascending (014589) hexachord and ends on a descent of that hexachord's self-complement. The second complete sequence is the first note of each Glied counted in sequence, or [017683te5492], which seems to have been a conscious decision on Stockhausen's part as the first four members of the first hexachord are related to the first four of the second by T_{10}, while the final two members of the first hexachord are related by T_1 to the final two of the second. The third complete sequence is the first twelve notes of the initial gestalt, [015794632e8t]. Given that there are 78 total pitch classes distributed across the twelve Glieder, a number not evenly divisible by 12, it's possible that Stockhausen had no intention of composing the rest of the gestalt out of complete aggregates beyond its beginning and end.

Given that the nature of each Glied is to distinguish itself in pitch-class content from the one prior to the furthest extent possible, with the previously mentioned exception of no. 7, it shouldn't be a surprise that each Glied shares many of the same pitch classes in common with both the Glied directly above the one prior, and the one directly below the next. Moreover, those pitch classes that are retained from one to the next as the Glieder grow in size tend to be grouped in clusters, making it likely that Stockhausen was conscious of dividing the initial gestalt into motivic subcomponents, which I show in Figure 7.

These motivic subcomponents develop and expand with each recurrence, and may be identified by an initial seed. Between Glied no. 1 and no. 10, there are three such subcomponents, which I label Motifs (015), (679), and (48t). The seed for motif (015) appears as the combination of Glieder nos. 1 and 2, and is distinguished by repeated alternation between pcs 1 and 5, preceded by pc0. The second instance appears in Glied no. 5, initially substituting pc4 for pc5 and inserting a pc3 in between them. The third instance in Glied no. 7 inserts pc3 between pcs 0 and 5. The motif undergoes a more drastic change in Glied no. 9, with pc1 appearing as acciaccature to pc11, and the alternating gesture transformed into a tremolo between pcs 11 and 3. By the fifth instance in Glied no. 10, Motif (015) is only identifiable by a single note each of pcs 5, 1, and 11. The same flexible manner of pitch class distribution may be seen in the other two motifs. Motifs (679) and (48t) may be understood, respectively, as general clusters of pcs 6, 7, and 9 with the occasional pc11, and pcs 4, 8 and 10 with the occasional pc2.

The increasingly disjoint nature of the later Glieder reaches a maximum point at no. 11, which is characterized by its staccato notes and large leaps. The melody includes a sequence [4e6183] that contains five ic7s in a row, which is unusual; the only other instances of consecutive interval classes found in the gestalt involve two in a row at most. It is possible that Stockhausen composed this lead melody starting from both ends, found himself at Glied no. 11 without substantial room to develop another motif any further, and chose to enlist a sensible, non-motivic gesture instead with the remaining pitch classes at his disposal.

After completing the second and third movements of *Spiel*, Stockhausen abandoned the first movement that would later be christened as *Formel* due to its different artistic style and compositional strategy. Sandwiched between the percussive, pointillist sounds of both *Kreuzspiel* and the now two-movement *Spiel*, the thematic motifs of *Formel* were heard by the budding composer as an unwelcome detour in the linear arc of his compositional development. This is understandable, as the vanguard of serialism at the time, including the likes of Olivier Messiaen and Pierre Boulez, were pushing further towards increased systematization of all parameters of music.

It's possible that Stockhausen was diverted from this path of total serialism by his stint in the electronic studios beginning in 1952, immediately after the composition of *Formel*. It shouldn't come as a surprise that Boulez withdrew many of his early pieces: any composer writing scores for live performance up until that time simply had little recourse to hear his or her pieces as physical sound until well after their completion, and serialist composers faced the additional hurdle of having to communicate unprecedented musical idioms to inexperienced and sometimes unsympathetic performers. By contrast, electronic technology allowed Stockhausen to experience the sounds he desired immediately upon their conception, granting him total control over every undertaking.

This experience allowed him to absorb two lessons early on. First, it is not necessarily an advantage for a composer to possess total control, as it can so easily lead to lifeless works. It is difficult for human thought processes to envision and project a dynamic character across all magnitudes of scale. For example, much effort

can be expended into creating well-rounded elements, yet paradoxically, the sum of those elements taken as a whole will not itself be well-rounded. The second lesson is that human performers can never hope to achieve the level of perfection proven possible with electronic media—nor should they, in light of that first lesson. By 1956, then, Stockhausen was contemplating ways to incorporate the inevitable inaccuracy of live performance into his works; in his article "…How Time Passes…" written that year, he describes these degrees of imprecision as "time-fields," which may themselves be serialized and arranged through statistical means.[7]

For the remainder of his life, Stockhausen supplemented his serialist understanding with statistical and aleatoric approaches. From his vantage, the cooling of friendship in later years between Boulez and John Cage due to their ideological differences surely must have seemed absurd. Total serialism and total chance are, after all, simply opposite means to the same end: they allow the composer to focus more on being human by reducing the kind of grueling busywork that most requires trying not to be. With this insight, the mature Stockhausen was finally able to recognize that *Formel*, with its jutting melodic fragments so uneven in constituency against the finer grains of its pointillist neighbors, stood all along as a perfectly fitting example of what serialism was always meant to accomplish.

[7] Karlheinz Stockhausen, "…How Time Passes…", trans. Cornelius Cardew, *Die Reihe* 3 (1959): 29-39.

Bibliography

Coenen, Alcedo. "Stockhausen's Paradigm: A Survey of His Theories." *Perspectives of New Music* 32, no. 2 (Summer 1994): 200-225.

Harvey, Jonathan. *The Music of Stockhausen: An Introduction*. Berkeley: University of California Press, 1975.

Kurtz, Michael. *Stockhausen: A Biography*. Translated by Richard Toop. London: Faber and Faber, 1992.

Maconie, Robin. *The Works of Karlheinz Stockhausen*. London: Oxford University Press, 1976.

Morgan, Robert P. "Stockhausen's Writings on Music." *The Musical Quarterly* 61, no. 1 (January 1975): 1-16.

Stockhausen, Karlheinz. *Formel für Orchester*. Vienna: Universal Edition, 1974.

_____. "…How Time Passes…" Translated by Cornelius Cardew. *Die Reihe* 3 (1959): 10-40.

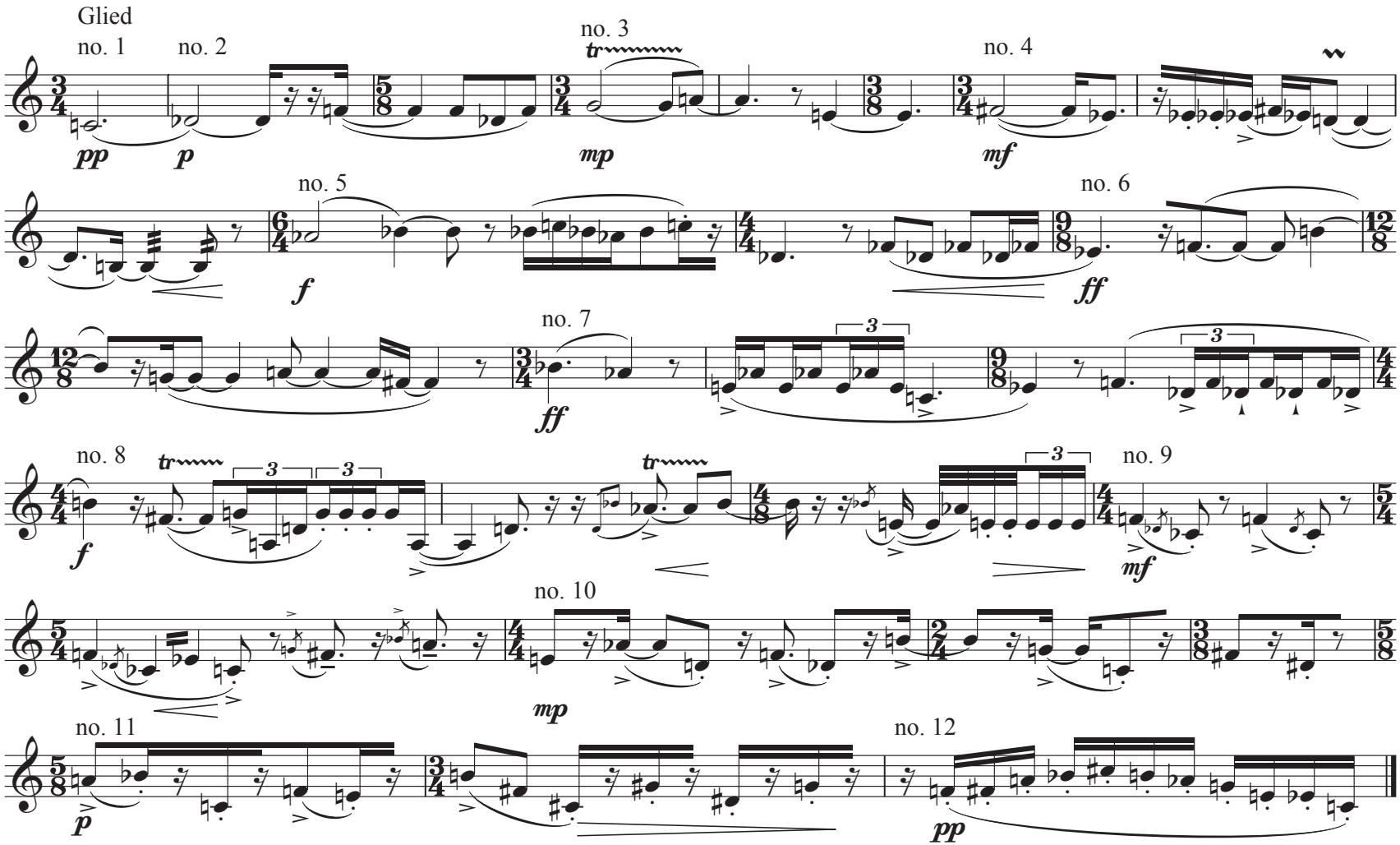

Figure 1. Initial gestalt played by vibraphone

Glied	no. of notes	note duration	time signature in initial gestalt	duration of Glied in eighth notes
no. 1	1	× 𝅗𝅥. =	3 ♩	6
no. 2	2	× 𝅗𝅥 + ♪. =	3 ♩ + 5 ♪	11
no. 3	3	× 𝅗𝅥 + ♪ =	3 ♩ + 3 ♩ + 3 ♪	15
no. 4	4	× 𝅗𝅥 + 𝅘𝅥𝅯 =	3 ♩ + 3 ♩ + 3 ♩	18
no. 5	5	× 𝅗𝅥 =	6 ♩ + 4 ♩	20
no. 6	6	× ♩.. =	3 ♩. + 4 ♩.	21
no. 7	7	× ♩. =	2 ♩. + 1 ♩. + 2 ♪. + 2 ♩. + 3 ♪	21
no. 8	8	× ♩ + 𝅘𝅥𝅯 =	4 ♩ + 4 ♩ + 4 ♪	20
no. 9	9	× ♩ =	4 ♩ + 5 ♩	18
no. 10	10	× ♪. =	4 ♩ + 2 ♩ + 3 ♪	15
no. 11	11	× ♪ =	5 ♪ + 3 ♩	11
no. 12	12	× 𝅘𝅥𝅯 =	3 ♩	6

Figure 2. Note durations and time signatures of each Glied

1	2	3	4	5	6	7	8	9	10	11	12
1											
2	1										
3	2	1									
4	3	2	1								
5	4	3	2	1							
6	5	4	3	2	1						
7	6	5	4	3	2	1					
8	7	6	5	4	3	2					
9	8	7	6	5	4	3					
10	9	8	7	6	5	4					
11	10	9	8	7	6	5					
12	11	10	9	8	7	6					
	12	11	10	9	8	7					
		12	11	10	9	8					
			12	11	10	9					
				12	11	10					
					12	11					
						12					
1											

Figure 3. Distribution of Glieder throughout entire piece

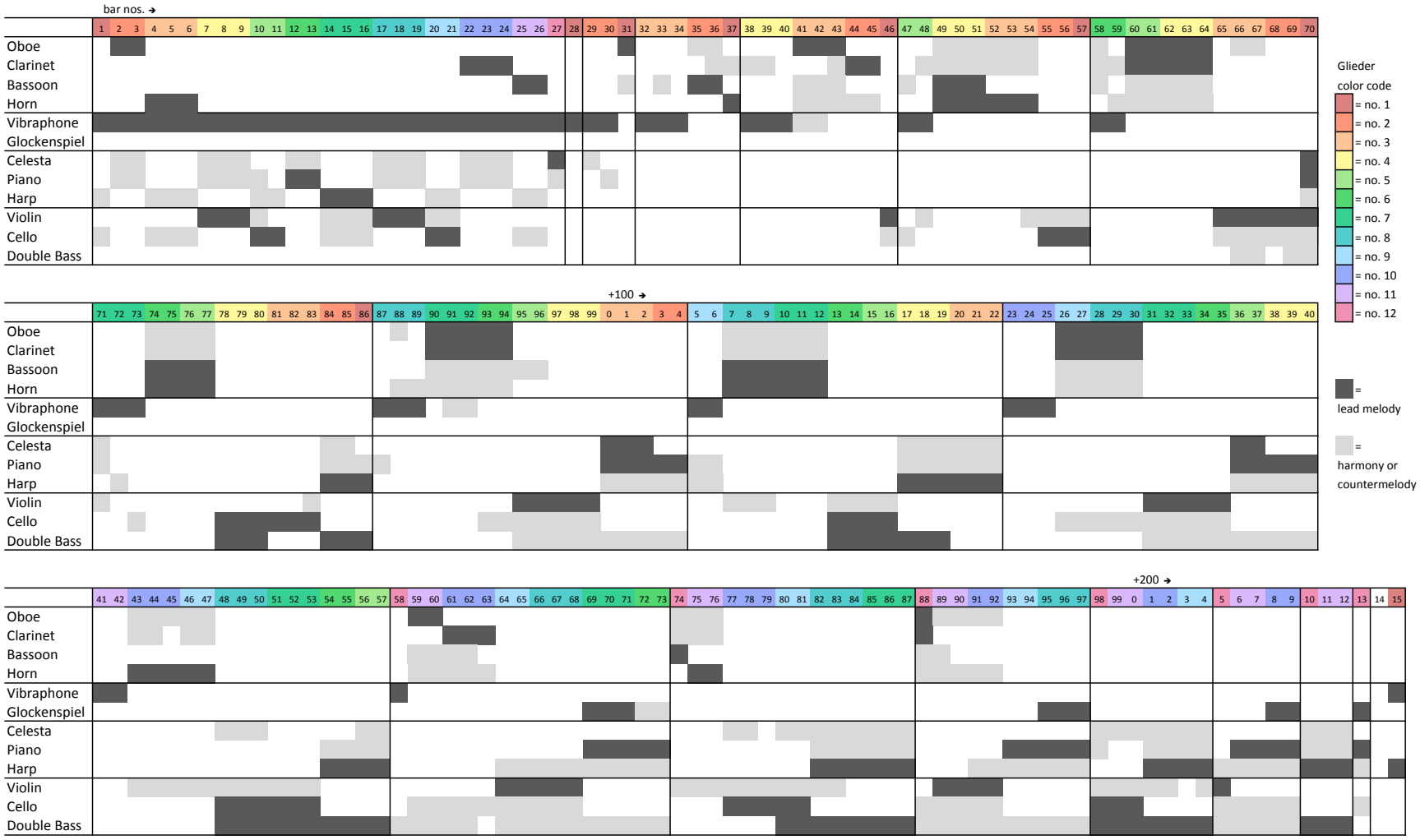

Figure 4. Visual map of entire piece

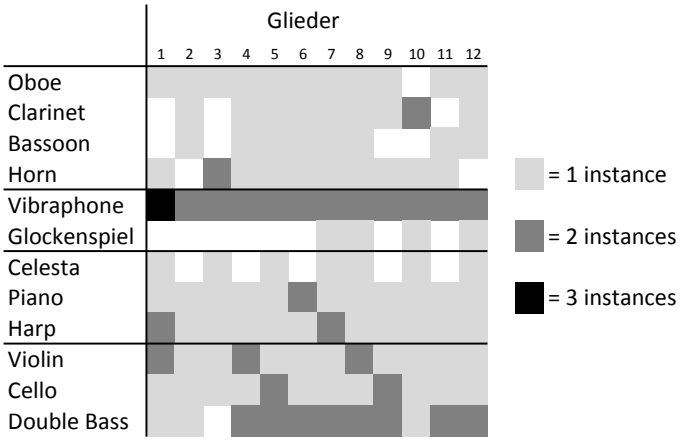

Figure 5. Number of instances of each Glied per instrument

Glied	pitch class content												pitch class order in initial gestalt											
no. 1	0												0											
no. 2		1				5							1	5										
no. 3					4			7		9			7	9	4									
no. 4			2	3			6					e	6	3	2	e								
no. 5	0	1			4				8		t		8	t	0	1	4							
no. 6				3		5	6	7		9		e	3	5	e	7	9	6						
no. 7	0	1		3	4	5			8		t		t	8	4	0	3	5	1					
no. 8			2		4		6	7	8	9	t	e	e	6	7	9	2	t	8	4				
no. 9	0	1		3		5	6	7		9	t	e	5	1	e	3	0	7	6	t	9			
no. 10	0	1	2	3	4	5	6	7	8			e	4	8	2	5	1	e	7	0	6	3		
no. 11	0	1		3	4	5	6	7	8	9	t	e	9	t	0	5	4	e	6	1	8	3	7	
no. 12	0	1	2	3	4	5	6	7	8	9	t	e	2	5	6	9	t	1	e	8	7	4	3	0

Figure 6. Pitch class content and order for each Glied

Glied	pitch class order in initial gestalt											
no. 1	0											
no. 2	1	5										
no. 3	7	9	4									
no. 4	6	3	2	e								
no. 5	8	t	0	1	4							
no. 6	3	5	e	7	9	6						
no. 7	t	8	4	0	3	5	1					
no. 8	e	6	7	9	2	t	8	4				
no. 9	5	1	e	3	0	7	6	t	9			
no. 10	4	8	2	5	1	e	7	0	6	3		
no. 11	9	t	0	5	4	e	6	1	8	3	7	
no. 12	2	5	6	9	t	1	e	8	7	4	3	0

= Motif (015)

= Motif (679)

= Motif (48t)

= ic7 sequence

= hexatonic sequence

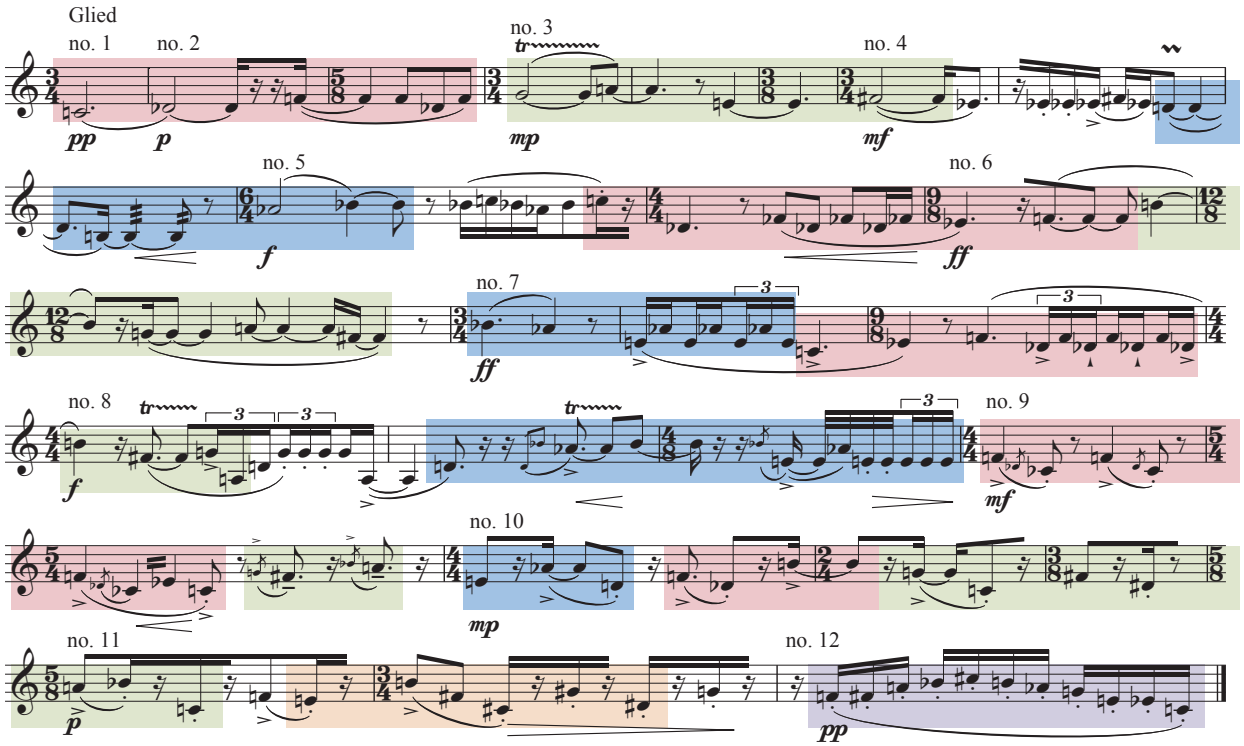

Figure 7. Division of initial gestalt into motivic subcomponents